MATH1520AB 2021-22 Quiz 8 (week 12)

Full marks: 10 marks

Time allowed: 15 minutes

1. Evaluate each of the following definite integrals.

   (a) $\int_1^9 (e^{2x} + \sqrt{x})dx$

   (b) $\int_{-2}^2 (|x+2| + |x-1|)dx$

   (c) $\int_4^8 \frac{1}{x^2 + 2x - 8}dx$

   Answer.

   (a) $\int_1^9 (e^{2x} + \sqrt{x})dx = \left[\frac{e^{2x}}{2} + \frac{2x^{\frac{3}{2}}}{3}\right]_1^9 = (\frac{e^{18}}{2} + 18) - (\frac{e^2}{2} + \frac{2}{3}) = \frac{e^{18}}{2} - \frac{e^2}{2} + \frac{52}{3}$

   (b) $\int_{-2}^2 (|x+2|+|x-1|)dx = \int_{-2}^1 [(x+2)-(x-1)]dx + \int_1^2 [(x+2)+(x-1)]dx = \int_{-2}^1 3dx + \int_1^2 (2x+1)dx = \left[3x\right]_{-2}^1 + \left[x^2 + x\right]_1^2 = 9 + 4 = 13$

   (c) $\int_4^8 \frac{1}{x^2 + 2x - 8}dx = \frac{1}{6}\int_4^8 \frac{1}{x-2} - \frac{1}{x+4}dx = \frac{1}{6}\left[\ln|x-2| - \ln|x+4|\right]_4^8 = \frac{1}{6}\left[(\ln 6 - \ln 12) - (\ln 2 - \ln 8)\right] = \frac{\ln 2}{6}$

2. Find the derivative $f'(x)$ of each of the following functions.

   (a) $f(x) = \int_1^{x^2} t^3 \ln(t^4 + 12)dt$

   (b) $f(x) = \int_{\sqrt{x}}^{x^3} (e^{t^2} - 2t)dt$

   Answer.

   (a) $f'(x) = (x^2)^3 \ln[(x^2)^4 + 12] \times (2x) = 2x^7 \ln(x^8 + 12)$

   (b) $f'(x) = [e^{(x^3)^2} - 2x^3](3x^2) - [e^{(\sqrt{x})^2} - 2\sqrt{x}](\frac{1}{2\sqrt{x}}) = 3x^2 e^{x^6} - \frac{e^x}{2\sqrt{x}} - 6x^5 + 1$